

4766

Mark Scheme

January 2013

Question		Answer	Marks	Guidance										
1	(i)	Positive	B1 [1]	CAO										
1	(ii)	Mean = 5.064 allow 5.1 with working 126.6/25 or 5.06 without SD = 1.324 allow 1.3 with working or 1.32 without	B1 B2 [3]	Allow B1 for RMSD = 1.297 or var = 1.753 or MSD = 1.683 Also allow B1 for $S_{xx} = 42.08$ or for $\Sigma x^2 = 683$ SC1 for both mean = 50.64 and SD = 13.24 (even if over-specified)										
1	(iii)	$\bar{x} - 2s = 5.064 - 2 \times 1.324 = 2.416$ $\bar{x} + 2s = 5.064 + 2 \times 1.324 = 7.712$ So there is an outlier.	B1FT M1 A1FT E1 [4]	FT their mean and sd for $\bar{x} + 2s$ but withhold final E mark if their limits mean that there are no outliers. For upper limit Incorrect statement such as 7.6 and 8.1 are outliers gets E0 Do not award E1 if calculation error in upper limit For use of quartiles and IQR $Q_1 = 3.95$; $Q_3 = 6.0$; IQR = 2.05 $3.95 - 1.5(2.05)$ gets M1 Allow other sensible definitions of quartiles $6.0 + 1.5(2.05)$ gets M1 Limits 0.875 and 9.075 So there are no outliers NB do not penalise over-specification here as not the final answer but just used for comparison. FT from SC1										
2	(i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>r</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>$P(X=r)$</td> <td>$3k$</td> <td>$8k$</td> <td>$15k$</td> <td>$24k$</td> </tr> </table> <p>$3k + 8k + 15k + 24k = 1$</p> <p>$k = 0.02$</p>	r	2	3	4	5	$P(X=r)$	$3k$	$8k$	$15k$	$24k$	B1 M1 A1 [3]	For correct table (ito k or correct probabilities 0.06, 0.16, 0.30, 0.48) or $k = 1/50$ (with or without working) For their four multiples of k added and =1. Allow M1A1 even if done in part (ii) – link part (ii) to part (i)
r	2	3	4	5										
$P(X=r)$	$3k$	$8k$	$15k$	$24k$										

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2	(ii)	$E(X) = (2 \times 0.06) + (3 \times 0.16) + (4 \times 0.30) + (5 \times 0.48) = 4.2$ <p>or 21/5</p> $E(X^2) = (4 \times 0.06) + (9 \times 0.16) + (16 \times 0.30) + (25 \times 0.48) = 18.48$ $\text{Var}(X) = 18.48 - 4.2^2$ $= 0.84 = 21/25$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>For Σrp (at least 3 terms correct Provided 4 reasonable probabilities seen.</p> <p>cao</p> <p>For $\Sigma r^2 p$ (at least 3 terms correct)</p> <p>dep for – their $E(X)^2$</p> <p>FT their $E(X)$ provided $\text{Var}(X) > 0$ (and of course $E(X^2)$ is correct)</p>	<p>If probs wrong but sum = 1 allow full marks here. If sum $\neq 1$ allow max M1A0M1 M0A0 (provided all probabilities between 0 and 1)</p> <p>Or ito k NB $E(X) = 210k$, $E(X^2) = 924k$ gets M1A0M1M0A0. $E(X) = 210k$, $\text{Var}(X) = 924k - (210k)^2$ gets M1A0M1M1A0.</p> <p>Use of $E(X - \mu)^2$ gets M1 for attempt at $(x - \mu)^2$ should see $(-2.2)^2$, $(-1.2)^2$, $(-0.2)^2$, 0.8^2, (if $E(X)$ wrong FT their $E(X)$) (all 4 correct for M1), then M1 for $\Sigma p(x - \mu)^2$ (at least 3 terms correct with their probabilities)</p> <p>Division by 4 or other spurious value at end gives max M1A1M1M1A0, or M1A0M1M1A0 if $E(X)$ also divided by 4.</p> <p>Unsupported correct answers get 5 marks</p>
3	(i)	$P(L \cap W) = P(L W) \times P(W) = 0.4 \times 0.07 = 0.028$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>For $P(L W) \times P(W)$</p> <p>cao</p>	

Question		Answer	Marks	Guidance
3	(ii)		<p>B1 For two labelled intersecting circles</p> <p>B1 For at least 2 correct probabilities.</p> <p>B1 For remaining probabilities</p> <p>[3]</p>	<p>FT their 0.028 provided < 0.038</p>
3	(iii)	<p>$P(L \cap W) = 0.028, P(L) \times P(W) = 0.038 \times 0.07 = 0.00266$</p> <p>Not equal so not independent</p>	<p>M1 For correct use of $P(L) \times P(W)$ If $P(L)$ wrong, max M1A0E0. No marks if $P(W)$ wrong</p> <p>A1 For 0.00266 E1* Allow 'they are dependent' Do not award E1 if $P(L \cap W)$ wrong</p> <p>[3]</p>	<p>Or EG $P(L W) = 0.4, P(L) = 0.038$ Not equal so not independent M1 is for comparing with some attempt at numbers $P(L W)$ with $P(L)$, A1 for 0.038 If $P(L)$ wrong, max M1A0E0</p>
4	(i)	$\binom{11}{3} = 165$	<p>M1 Seen</p> <p>A1 Cao</p> <p>[2]</p>	

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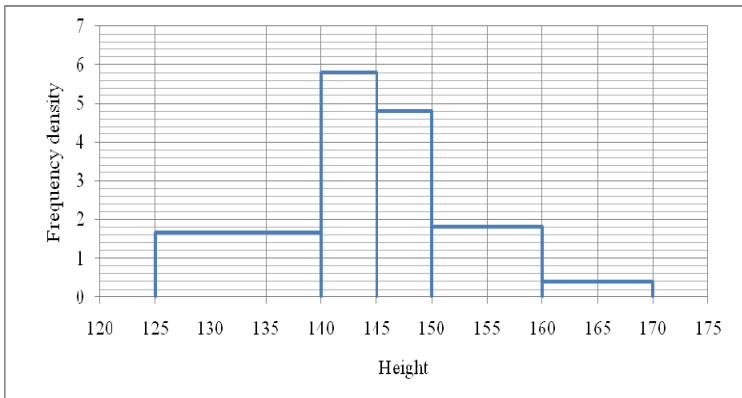
Question	Answer	Marks	Guidance	
4 (ii)	$\frac{\binom{5}{2} \times \binom{6}{1}}{\binom{11}{3}} + \frac{\binom{5}{3} \times \binom{6}{0}}{\binom{11}{3}} = \frac{60}{165} + \frac{10}{165} = \frac{70}{165} = \frac{14}{33} = 0.424$ <p>Alternative</p> $1 - P(1 \text{ or } 0) = 1 - 3 \times \frac{5}{11} \times \frac{6}{10} \times \frac{5}{9} - \frac{6}{11} \times \frac{5}{10} \times \frac{4}{9}$ $= 1 - \frac{5}{11} - \frac{4}{33} = \frac{14}{33}$ <p>M1 for $1 - P(1 \text{ or } 0)$, M1 for first product, M1 for $\times 3$, M1 for second product, A1</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>For intention to add correct two fractional terms</p> <p>For numerator of first term</p> <p>For numerator of sec term</p> <p>Do not penalise omission of $\binom{6}{0}$</p> <p>For correct denominator</p> <p>cao</p>	<p>Or</p> <p>For attempt at correct two terms</p> <p>For prod of 3 correct fractions = 4/33</p> <p>For whole expression ie</p> $3 \times \frac{5}{11} \times \frac{4}{10} \times \frac{6}{9} \left(= \frac{4}{11} \right) (= 3 \times 0.1212\dots)$ <p>For attempt at $\frac{5}{11} \times \frac{4}{10} \times \frac{3}{9} \left(= \frac{2}{33} \right)$</p> <p>cao</p> <p>Use of binomial can get max first M1</p>
5 (i)	$\left(\frac{5}{6}\right)^2 \times \frac{1}{6} = \frac{25}{216} (= 0.116)$	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For 5/6 (or $1 - 1/6$) seen</p> <p>For whole product</p> <p>cao</p>	<p>If extra term or whole number factor present give M1M0A0</p> <p>Allow 0.12 with working</p>
5 (ii)	$1 - \left(\frac{5}{6}\right)^{10} = 1 - 0.1615 = 0.8385$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>For $(5/6)^{10}$ (without extra terms)</p> <p>cao</p>	<p>Allow 0.838 or 0.839 without working and 0.84 with working.</p> <p>For addition $P(X = 1) + \dots + P(X = 10)$ give M1A1 for 0.84 or better, otherwise M0A0</p>

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Question		Answer	Marks	Guidance
6	(i)	$4 + \frac{1}{2} \text{ of } 18 = 4 + 9 = 13$	M1 A1 [2]	For $\frac{1}{2}$ of 18 cao 13/100 gets M1A0
6	(ii)	(Median) = 50.5 th value $\text{Est} = 140 + \left(\frac{25.5}{29}\right) \times 5 \quad \text{or} = 140 + \left(\frac{50.5 - 25}{54 - 25}\right) \times 5$ $= 144.4$	M1 M1 A1 [3]	For 50.5 seen For attempt to find this value SC2 for use of 50 th value leading to $\text{Est} = 140 + (25 / 29 \times 5) = 144.3$ (SC1 if over-specified) $\text{or Est} = 145 - \left(\frac{3.5}{29}\right) \times 5 = 144.4$ NB no marks for mean = 144.35 NB Watch for over-specification

Question	Answer	Marks	Guidance																								
6 (iii)	<table border="1" data-bbox="369 239 1108 518"> <thead> <tr> <th>Height</th> <th>Frequency</th> <th>Group width</th> <th>Frequency density</th> </tr> </thead> <tbody> <tr> <td>$125 \leq x \leq 140$</td> <td>25</td> <td>15</td> <td>1.67</td> </tr> <tr> <td>$140 < x \leq 145$</td> <td>29</td> <td>5</td> <td>5.80</td> </tr> <tr> <td>$145 < x \leq 150$</td> <td>24</td> <td>5</td> <td>4.80</td> </tr> <tr> <td>$150 < x \leq 160$</td> <td>18</td> <td>10</td> <td>1.80</td> </tr> <tr> <td>$160 < x \leq 170$</td> <td>4</td> <td>10</td> <td>0.40</td> </tr> </tbody> </table> 	Height	Frequency	Group width	Frequency density	$125 \leq x \leq 140$	25	15	1.67	$140 < x \leq 145$	29	5	5.80	$145 < x \leq 150$	24	5	4.80	$150 < x \leq 160$	18	10	1.80	$160 < x \leq 170$	4	10	0.40	<p>M1</p> <p>A1</p> <p>G1</p> <p>W1</p> <p>H1</p>	<p>For fd's - at least 3 correct</p> <p>Accept any suitable unit for fd such as eg freq per cm. correct to at least one dp allow 1.66 but not 1.6 for first fd</p> <p>linear scales on both axes and label on vertical axis</p> <p>width of bars</p> <p>height of bars</p> <p>M1 can be also be gained from freq per 10 – 16.7, 58, 48, 18, 4 (at least 3 correct) or freq per 5 – 8.35, 29, 24, 9, 2 for all correct. If fd not explicitly given, M1 A1 can be gained from all heights correct (within one square) on histogram (and M1A0 if at least 3 correct)</p> <p>Linear scale and label on vertical axis IN RELATION to first M1 mark ie fd or frequency density or if relevant freq/10, etc (NOT eg fd/10). However allow scale given as fd×10, or similar Accept f/w or f/cw (freq/width or freq/class width) Can also be gained from an accurate key G0 if correct label but not fd's. Must be drawn at 125, 140 etc NOT 124.5 or 125.5 etc NO GAPS ALLOWED Must have linear scale. No inequality labels on their own such as $125 \leq S < 140$, etc but allow if a clear horizontal linear scale is also given. Ignore horizontal label.</p> <p>Height of bars – must be linear vertical scale. FT of heights dep on at least 3 heights correct and all must agree with their</p>
Height	Frequency	Group width	Frequency density																								
$125 \leq x \leq 140$	25	15	1.67																								
$140 < x \leq 145$	29	5	5.80																								
$145 < x \leq 150$	24	5	4.80																								
$150 < x \leq 160$	18	10	1.80																								
$160 < x \leq 170$	4	10	0.40																								

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			[5]	fds If fds not given and at least 3 heights correct then max M1A0G1W1H0 Allow restart with correct heights if given fd wrong (for last three marks only)												
6	(iv)	4 boys 0.6×15 = 9 girls So 5 more girls	M1 A1 A1 [3]	For 0.6×15 For 9 girls cao Or $45 \times 0.2 = 9$ (number of squares and 0.2 per square)												
6	(v)	Frequencies and midpoints for girls are <table border="1" style="margin-left: 20px;"> <tr> <td>Height</td> <td>132.5</td> <td>142.5</td> <td>147.5</td> <td>155</td> <td>167.5</td> </tr> <tr> <td>Frequency</td> <td>18</td> <td>23</td> <td>31</td> <td>19</td> <td>9</td> </tr> </table> So mean = $\frac{(132.5 \times 18) + (142.5 \times 23) + (147.5 \times 31) + (155 \times 19) + (167.5 \times 9)}{100}$ $= \frac{(2385) + (3277.5) + (4572.5) + (2945) + (1507.5)}{100}$ $= 146.9$ (Exact answer 146.875)	Height	132.5	142.5	147.5	155	167.5	Frequency	18	23	31	19	9	B1 For at least three frequencies correct B1 At least three midpoints correct M1 M1* Dep on M1 For attempt at $\sum xf$ For division by 100 A1 Cao NB Watch for over-specification [5]	No further marks if not using midpoints For sight of at least 3 xf pairs Allow answer 146.9 or 147 but not 150 NB Accept answers seen without working (from calculator) Use of 'not quite right' midpoints such as 132.49 or 132.51 etc can get B1B0M1M1A0
Height	132.5	142.5	147.5	155	167.5											
Frequency	18	23	31	19	9											

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7	(i)	(A)	$X \sim B(10, 0.35)$ $P(5 \text{ accessing internet}) = \binom{10}{5} \times 0.35^5 \times 0.65^5$ $= 0.1536$ OR from tables = $0.9051 - 0.7515 = 0.1536$	M1 M1 A1 OR M2 A1 [3]	or $0.35^5 \times 0.65^5$ For $\binom{10}{5} \times p^5 \times q^5$ cao For $0.9051 - 0.7515$ cao	With $p + q = 1$ Also for 252×0.0006094 Allow 0.15 or better NB 0.153 gets A0 See tables at the website http://www.mei.org.uk/files/pdf/formula_book_mf2.pdf
7	(i)	(B)	$P(X \geq 5) = 1 - P(X \leq 4)$ $= 1 - 0.7515$ $= 0.2485$	M1 A1 [2]	For 0.7515 cao	Accept 0.25 or better – allow 0.248 or 0.249 Calculation of individual probabilities gets B2 if fully correct 0.25 or better, otherwise B0.
7	(i)	(C)	$E(X) = np = 10 \times 0.35$ $= 3.5$	M1 A1 [2]	For 10×0.35 cao	If any indication of rounding to 3 or 4 allow M1A0

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7	(ii)	<p>Let $X \sim B(20, 0.35)$ Let p = probability of a customer using the internet (for population)</p> <p>$H_0: p = 0.35$</p>	<p>B1</p> <p>B1</p>	<p>For definition of p in context</p> <p>For H_0</p>	<p>Minimum needed for B1 is p = probability of using internet. Allow $p = P(\text{using internet})$ Definition of p must include word probability (or chance or proportion or percentage or likelihood but NOT possibility). Preferably as a separate comment. However can be at end of H_0 as long as it is a clear definition 'p = the probability of using internet', Do NOT allow 'p = the probability of using internet is different'</p> <p>Allow $p=35\%$, allow only p or θ or π or ρ. However allow any single symbol <u>if defined</u> (including x) Allow $H_0 = p=0.35$, Allow $H_0: p=7/20$ or $p=35/100$ Allow NH and AH in place of H_0 and H_1 Do not allow $H_0: P(X=x) = 0.35$ Do not allow $H_0: =0.35, =35\%, P(0.35), p(x)=0.35, x=0.35$ (unless x correctly defined as a probability) Do not allow H_0 and H_1 reversed For hypotheses given in words allow Maximum BOB1B1 Hypotheses in words must include probability (or chance or proportion or percentage) and the figure 0.35 oe Thus eg $H_0: p(\text{using internet}) = 0.35$, $H_1: p(\text{using internet}) \neq 0.35$ gets BOB1B1</p>

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	<p>$H_1: p \neq 0.35$</p> <p>H_1 has this form because the test is to investigate whether the proportion is different, (rather than lower or higher). $P(X \geq 10)$</p> <p>$= 1 - 0.8782 = 0.1218$</p> <p>$> 2.5\%$</p> <p>So not significant. Conclude that there is not enough evidence to indicate that the probability is different. (Must state 'probability', not just 'p')</p> <p>ALTERNATIVE METHOD FOR FINAL 5 MARKS</p> <p>Critical region method LOWER TAIL $P(X \leq 2) = 0.0121 < 2.5\%$ $P(X \leq 3) = 0.0444 > 2.5\%$</p> <p>UPPER TAIL $P(X \geq 11) = 1 - P(X \leq 10) = 1 - 0.9468 = 0.0532 > 2.5\%$ $P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.9804 = 0.0196 < 2.5\%$</p>	<p>B1</p> <p>E1</p> <p>B1</p> <p>B1*</p> <p>M1* dep A1* E1* dep on A1</p> <p>B1</p> <p>B1</p>	<p>For H_1</p> <p>For notation $P(X \geq 10)$ or $P(X > 9)$ or $1 - P(X \leq 9)$ (as long as no incorrect notation)</p> <p>For 0.1218 Allow 0.12</p> <p>For comparison with 2.5%</p> <p>For either probability</p> <p>For either probability</p>	<p>Allow '$p < 0.35$ or $p > 0.35$' in place of $p \neq 0.35$ Do not allow if H_1 wrong.</p> <p>This mark may be implied by 0.1218 as long as no incorrect notation. No further marks if point probs used - $P(X = 10) = 0.0686$ (do not even give the notation mark for correct notation) DO NOT FT wrong H_1, but see extra notes Or for $1 - 0.8782$ Indep of previous mark</p> <p>Allow 'accept H_0' or 'reject H_1' Must include 'sufficient evidence' or something similar such as 'to suggest that' ie an element of doubt either in the A or E mark.</p> <p>Do not insist on correct notation as candidates have to work out two probabilities for full marks. If only upper tail of CR given (or only upper tail justified), allow max 4/5 for final 5 marks.</p>

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		<p>So critical region is $\{0,1,2,12,13,14,15,16,17,18,19,20\}$</p> <p>So not significant Conclude that there is not enough evidence to indicate that the probability is different.</p>	<p>M1* dep</p> <p>A1* E1* dep on A1</p> <p>[9]</p>	<p>cao dep on at least one correct comparison with 2.5%</p> <p>No marks if CR not justified Condone $\{0,1,2, 12, \dots 20\}$, $X \leq 2$, $X \geq 12$, oe but not $P(X \leq 2)$ etc</p> <p>NB If CR found correctly then $P(X = 10)$ subsequently found but cand says '10 not in CR' then allow up to all last five marks. If do not say '10 not in CR' allow none of last five marks</p>
7	(iii)	<p>$0.0022 < 2.5\%$ So reject H_0, Significant.</p> <p>Conclude that there is enough evidence to indicate that the probability is different.</p>	<p>B1</p> <p>E1* dep</p> <p>[2]</p>	<p>For either reject H_0 or significant, dep on correct comparison</p> <p>Dep on good attempt at correct hypotheses in part (ii)</p> <p>If they have $H_1: p > 0.35$, allow SC1 if all correct including comparison with 5%.</p>

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APPENDIX**NOTE RE OVER-SPECIFICATION OF ANSWERS**

If answers are grossly over-specified, deduct the final answer mark in every case. Probabilities should also be rounded to a sensible degree of accuracy. In general final non-probability answers should not be given to more than 4 significant figures. Allow probabilities given to 5 sig fig.

Additional notes re Q7 part iiComparison with 97.5% method

If 97.5% seen anywhere then

B1 for $P(X \leq 9)$

B1 for 0.8782

M1* for comparison with 97.5% dep on second B1

A1* for not significant oe

E1*

Smallest critical region method:

Smallest critical region that 10 could fall into is {10,11,12,13,14,15, 16, 17, 18,19,20} gets B1 and has size 0.1218 gets B1, This is > 2.5% gets M1*, A1*, E1* as per scheme

NB These marks only awarded if 10 used, not other values.

Use of k method with no probabilities quoted:

This gets zero marks.

Use of k method with one probability quoted:

Mark as per scheme

Line diagram method and Bar chart method

No marks unless correct probabilities shown on diagram, then mark as per scheme.

Upper tailed test done with $H_1: p > 0.35$

Hyp gets max B1B1B0E0

If compare with 5% give SC2 for $P(X \geq 10) = 1 - 0.8782 = 0.1218 > 5\%$ and SC1 for final conclusion (must be 'larger than' not 'different from')

If compare with 2.5% no further marks BOBOM0A0E0

Lower tailed test done with $H_1: p < 0.35$

No marks out of last 5.